

A decorative graphic on the left side of the cover features several playing cards. Two cards are prominently displayed: the Ace of Hearts and the Ace of Diamonds. They are surrounded by various suits including hearts, clubs, and diamonds, some in red and some in black, scattered around them. The cards have a white face with a gold border.

JULIAN
LADERMAN

USEFUL
PROBABILITY
FOR BRIDGE
PLAYERS



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MASTER POINT PRESS • TORONTO

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DEDICATION

To Antoinette, my great partner in life

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Several other friends and bridge partners also proofread versions of this book: Mich Araten, Farley Mawyer, Amalia Ottemberg, Sandy Prosnitz, and Art Seelenfreund. Cliff Nebel provided me with copies of several old journal articles.

I want to thank Antoinette Blum for her patience and support. I particularly appreciate her willingness to proofread the book since she does not play bridge nor has she ever taken a probability course. Her reading experience must have been painful. Even more so since I refer to her at times in the book.

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Preface

In 2009, Master Point Press published my book *A Bridge to Inspired Declarer Play*. The original manuscript contained a thirty-five-page Appendix on the probability of bridge. Master Point Press correctly objected to some of the harder mathematics, since they thought it would discourage many readers from ever getting to some of the useful material that appeared towards the end of the Appendix. After some discussion, we agreed on a Solomonic solution: I would cut the Appendix into two. The first part would contain the more useful techniques and the second the more difficult underlying mathematics. The titles of the two appendices are Appendix 1: Everything You Ever Wanted to Know about Probability and Appendix 2: Much More Than You Ever Wanted to Know about Probability.

The appendices in that 2009 book were the basis for *The New York Times* (Phillip Alder) quote on the back cover of this book. They have been greatly expanded, with their titles carried over for nostalgic purposes, to Part II and Part III of this book.

The rest of this Preface consists of my apologies.

Apology 1: I am assuming readers have virtually no prior knowledge of mathematics – just some arithmetic with fractions. I fear some readers may be insulted. While reading this book, if you feel demeaned, please do not hesitate to roll your eyes.

Apology 2: I am assuming readers have played at least a thousand hands of bridge (roughly forty afternoons) but I rarely assume any specific bridge knowledge beyond that playing experience. Therefore, at times, some readers may feel insulted. Again, please feel free to roll your eyes.

Apology 3: I often steal material (I prefer ‘borrow’) from my earlier books and articles. Here, as already mentioned, the greatest victim of this theft is *A Bridge to Inspired Declarer Play*. About 15% of the book you are holding is taken from it. My other books were not ignored – approximately 5% of this book was stolen from

Bumblepuppy Days: The Evolution from Whist to Bridge, as well as 3% in total from my other three books.

So, if you have previously read those books, and find that some material looks familiar, you are correct.

Apology 4: The gender problem of using the pronoun ‘he’ or ‘she’ is troublesome for writers. I certainly do not like switching between them. Since the bridge world has more women than male participants, I want to inspire the male minority so I will use ‘he’ rather than ‘she’. Besides, this saves me from typing an additional letter. If you are offended, I apologize.

Apology 5: I suffer from the UDBH syndrome. If you are not familiar with the ailment, it stands for Uncontrollably Describing Bridge History. The most troubling symptom is that it forces me to include at least a smidgen of bridge history whenever I write or give a talk about bridge. This was partially provoked by writing a book on bridge history in 2014, but I now realize that I have shown symptoms for more than three decades. If these inclusions trouble you, please remember that I cannot control myself.

I hope you will enjoy this book, find it informative, and not see any need for me to apologize for anything else.

Julian Laderman

PART 1

*The only thing we
can really be certain
of is that from birth
to death we are
completely bound by
the ever-present and
wholly inclusive laws
of probability that
govern every action
and event.*

Oswald Jacoby

TABLE TALK AT THE BRIDGE TABLE



Who Needs Bridge Probability?

Experience is a Great Teacher

Often at a bridge table an opponent who is aware of my prior professional life comments, 'I hated mathematics. I'm sure glad that playing bridge does not require any knowledge of mathematics.' I immediately look for the nearest Director and estimate the approximate time he needs to cover the distance to my table. This will dictate how lengthy a response I can provide before hearing 'Julian, stop talking! The round has been called.' Sadly, probably only for me, there is never enough time. This book provides me with the opportunity to give a full retort.

Probability is the mathematics of studying how likely an event is to occur. All experienced bridge players have a significant amount of knowledge in this field, whether they realize it or not. Let me demonstrate by asking you four quick questions:

- 1) Are you more likely to be dealt 14 high-card points (HCP) or 21 HCP?
- 2) Your partner opens 1♥. Is he more likely to hold five hearts or six hearts?
- 3) You open the bidding 1♠. Is your partner more likely to have three spades or a void in spades?
- 4) You open the bidding 1NT. Is your partner more likely to have 6 HCP or 1 HCP?

The first option was the correct answer to each of the four questions. Let's hope you just got a 100% on this probability test. You were using the experience you gained from playing thousands of bridge hands to estimate likelihood. Your performance was likely

far better than that of a person equipped with a Ph.D. in mathematics who has never played bridge. Be proud!

One way to measure the likelihood of an event occurring is to measure its relative frequency. This is obtained by simply recording how often an event occurs in a large number of trials.

$$\text{Relative frequency} = \frac{\text{number of times event occurs}}{\text{number of trials}}$$

Suppose a baseball player has had 423 official times at bat and has 132 hits. His batting average is obtained by dividing 132 by 423. The resulting value of 0.312 (or 31.2%) is very impressive and his agent will certainly use it when negotiating the player's next contract. The baseball world ignores the decimal point; the batting average is 312.

Even though you have not kept track of bridge hands* that you have played over the years, you were able to use your experience to answer the above questions. Of course, it would have been much more difficult to estimate the actual likelihood of each of the events rather than to compare the two.

For example, suppose you wish to know the likelihood of being dealt a hand where the longest suit has five or more cards. If you kept track of your next 1000 hands, and recorded how many had at least one suit with five or more cards, you would obtain an extremely accurate estimate of the true value. Please don't carry out this experiment – it would be foolish, since it is very easy to calculate the likelihood of the event. I won't leave you hanging: it is 65%.

In a very early bridge lesson, a beginner is told that the number 26 is special. With a combined 26 high-card points (HCP) a

* Bridge vocabulary is a little vague at times. Consider the words 'hand' and 'deal'. A bridge deal consists of all four bridge hands. The word hand can refer either to the thirteen cards a player is dealt or to all four hands. Sometimes the word 'hand' and 'deal' overlap in usage. At the end of a session, we often remind our partners to pick up 'hand records' so that they can be discussed over dinner. It would be more accurate to refer to those sheets as 'deal records'.

partnership has enough good cards that they can probably make a game in notrump or a major. Of course, any bridge player can construct freakish pairs of hands with a combined 30 HCP that cannot make game and pairs with a combined 20 HCP that can. The 26 HCP number is based on the experience of top players; this kind of guideline or maxim allows new players to learn bridge probability more quickly than by relying on their own personal experience. Our bidding systems are based on this and similar maxims. In Chapter 15, we will look at the maxims that relate to bidding and see how bidding systems are designed to communicate the information necessary for a player to employ these maxims.

Bridge players are armed with many probability-based maxims for both bidding and play. I cringe slightly when they are referred to as rules (or even laws) rather than guidelines. We will be looking at many of them throughout the book in order to learn when they are useful and when they may be misleading.

A one-sentence answer to the question posed in the title of this chapter: *You greatly need it, and you already know much more than you think.*

One Can Be Fooled by Experience

After extolling the virtues of intuition generated through experience, I must point out that it often can be quite misleading. Let me add one more question to the earlier four.

Is the next bridge hand you are dealt more likely to be:

♠ A 7 5 4 ♥ K 8 5 ♦ A 8 7 2 ♣ A 6

or ♥ A K Q J 10 9 8 7 6 5 4 3 2 and void in the other suits?

In fact, both are equally likely to be your next hand. It is tempting but wrong to believe that the first hand is more likely to occur. The first hand feels like a common hand whereas the second hand is extremely remarkable. If you were actually dealt the second hand, you would be telling all your friends, even your non-bridge playing friends, ‘You won’t believe the hand that I was dealt!’

GENERAL INTEREST

Beginners rely heavily on bridge maxims. Are they accurate? What is the mathematics behind them? *Useful Probability for Bridge Players* examines these questions. The emphasis here is on ‘useful’. This is not an academic tome, but a discussion of the aspects of probability that every bridge player needs to know and understand. Topics include suit splits, suit combinations, percentage plays, the Principle of Restricted Choice, choosing bidding systems/conventions and the application of probability to bidding decisions.

– Praise for *A Bridge to Inspired Declarer Play* –

“... [Laderman] gives the most comprehensible explanation of probabilities I can remember reading.”

The New York Times (Dec, 2009) Phillip Alder



JULIAN LADERMAN, Ph.D (New York) is a retired applied mathematics professor (CUNY) who specializes in making difficult concepts clear. He is the author of *A Bridge to Simple Squeezes* and *A Bridge to Inspired Declarer Play*, both ABTA Book of the Year award winners. His bridge history book, *Bumblepuppy Days*, won the IBPA Alan Truscott Memorial Award.

